Funkcionalni redovi

October 22, 2017

Funkcionalni red

$(f_n(x))_{n\in\mathbb{N}}$ je funkcionalni niz.

$\sum_{n=1}^{\infty} f_n(x)$ je funkcionalni red.

Primer

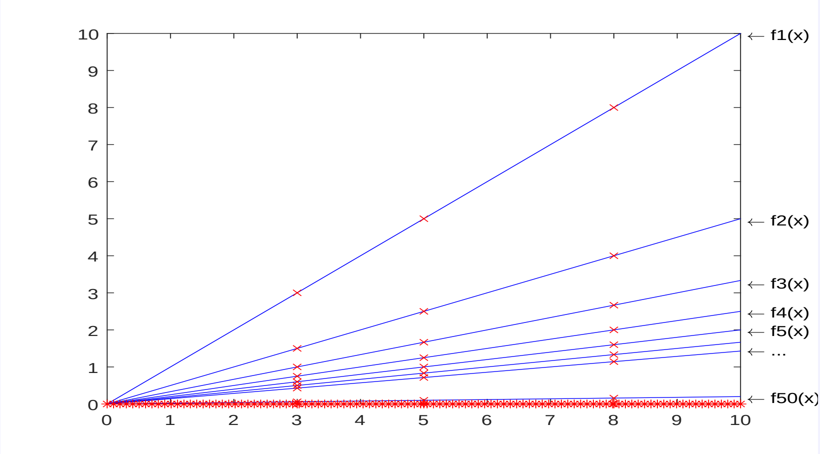
Za funkcionalni niz $f_n(x) = \frac{x}{n}, (x \in \mathbb{R}, n \in \mathbb{N})$, *odgovarajući funkcionalni red je* $\sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{x}{n} = \frac{x}{1} + \frac{x}{2} + \frac{x}{3} + \ldots$.

Podsećanje: Konvergencija funkcionalnog niza

$$f_n(x) = \frac{x}{n}, \qquad (x \in \mathbb{R}, n \in \mathbb{N})$$

$$\lim_{n\to\infty} f_n(x) = f(x) = 0 \text{ (funkcija! a ne broj)}$$

$$(\forall \varepsilon > 0)(\forall x \in I)(\exists N(\varepsilon, x))(\forall n > N(\varepsilon, x)) : |f_n(x) - f(x)| < \varepsilon$$

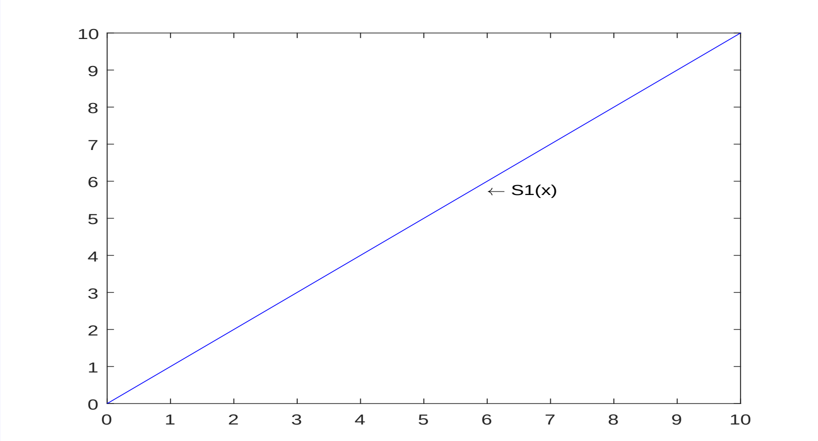


Konvergencija funkcionalnog reda

$$\sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{x}{n} = \frac{x}{1} + \frac{x}{2} + \frac{x}{3} + \ldots$$

Posmatramo niz parcijalnih suma $S_n(x) = f_1(x) + f_2(x) + \ldots + f_n(x)$.

$S_1(x) = \frac{x}{1}$

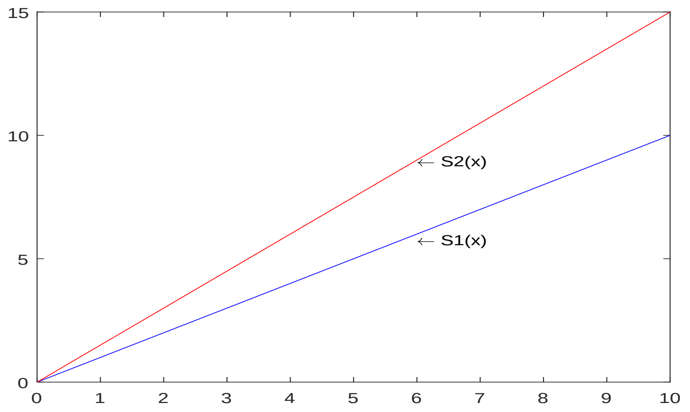


Konvergencija funkcionalnog reda

$$\sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{x}{n} = \frac{x}{1} + \frac{x}{2} + \frac{x}{3} + ...$$

Posmatramo niz parcijalnih suma $S_n(x) = f_1(x) + f_2(x) + ... + f_n(x)$.

$S_1(x) = \frac{x}{1},\ S_2(x) = \frac{x}{1} + \frac{x}{2} = \frac{3x}{2}$

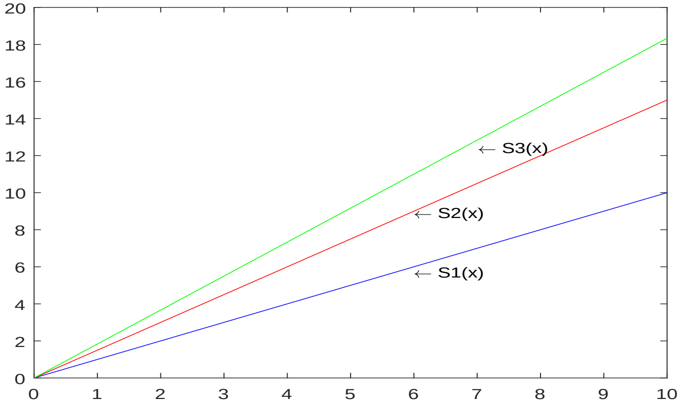


Konvergencija funkcionalnog reda

$$\sum_{n=1}^{\infty} f_n(x) = \sum_{n=1}^{\infty} \frac{x}{n} = \frac{x}{1} + \frac{x}{2} + \frac{x}{3} + \dots$$

Posmatramo niz parcijalnih suma $S_n(x) = f_1(x) + f_2(x) + \dots + f_n(x)$.

$S_1(x) = \frac{x}{1},\ S_2(x) = \frac{x}{1} + \frac{x}{2} = \frac{3x}{2},\ S_3(x) = \frac{x}{1} + \frac{x}{2} + \frac{x}{3} = \frac{11x}{6}$



Konvergencija funkcionalnog reda

$S_1(x) = \frac{x}{1}$, $S_2(x) = \frac{x}{1} + \frac{x}{2} = \frac{3x}{2}$, $S_3(x) = \frac{x}{1} + \frac{x}{2} + \frac{x}{3} = \frac{11x}{6}$ $S_4(x) = \frac{x}{1} + \frac{x}{2} + \frac{x}{3} + \frac{x}{4} = \frac{25x}{12}, \ldots$

Da li $\exists \lim_{n\to\infty} S_n(x)$?